

PART 1. Part 1 consists of 6 questions. Do your work and clearly write your answer in the space provided. No partial credit is awarded for this part of the test. (5 points each)

1. Set up, **but do not evaluate**, an integral for the area of the region bounded by $y = x^2$ and $y = 4$.

Answer: _____

2. Evaluate the improper integral, or show it diverges. $\int_0^{\infty} xe^{-x^2} dx$

Answer: _____

3. A spring of natural length 1.5 ft. requires a force of 10 lbs. to hold it stretched 0.5 ft past its natural length. How much work would need to be done to stretch the spring from its natural length to an additional 1.5 ft.?

Answer: _____

4. Use an appropriate comparison test to determine whether or not $\int_1^{\infty} \frac{1}{\sqrt{x^2-1}} dx$ converges or diverges.

Answer: _____

5. Is the area under the curve $y = \frac{1}{\sqrt{x}}$ from $x = 0$ to $x = 1$ finite or infinite? If finite, what is it?

Answer: _____

6. Set up, but **do not evaluate**, an integral that represents the length of the curve given by $x = t + \cos t$, $y = t - \sin t$, $0 \leq t \leq 2\pi$.

Answer: _____

Part 2. Part 2 consists of 5 problems worth 14 points apiece. Show all your work for full credit! Displaying only the final answer (even if correct) without the relevant steps is not enough.

Problem 1

Sketch the region of the plan bounded by the curve $y = \sqrt{x}$, $y = 6 - x$ and $y = 1$. Then use an integral to determine the area.

Problem 2

Use the method of slicing (disk method) to set up integrals complete with limits of integration, and in terms of just one variable, (but, **do not evaluate**) to find the volume of the solid generated by revolving the region bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the lines:

a. the x-axis

b. $y = 2$

Problem 3

3. Sketch the region bounded by $y = x^3$, $x = 2$, and the x -axis. Use cylindrical shells to find the volume of the solid of revolution generated by revolving this region about the y -axis.

Problem 4

- a. Find the length of the curve $y = \ln(\cos x)$, $0 \leq x \leq \frac{\pi}{3}$.
- b. Set up, but **do not evaluate** an integral that represents the area of the region enclosed by $y = 5x - x^2$ and the line $y = x$. Your integral should be complete with limits and in terms of just one variable.

Problem 5

Find the work done in pumping the water out of a circular cone of height $h=7m$ and radius $r=5m$. You may use the fact that the density of water is 1000 kg/m^3 and that the acceleration due to gravity is $9.8m/\text{sec}^2$. You can **just set up** the needed integral in terms of one variable and appropriate limits. **You do not need to integrate.**